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Example of the Couette iceform design model: flat plate iceformation

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Abstract—This paper presents an application of the Couette iceform design model. Two-dimensional iceformation over a cold flat plate is used to demonstrate the evolution theory approach to controlling the ice formation shaping process. Theoretical ice profiles reasonably matched experimental results. Optimal control parameter relationships for designing minimum energy dissipation shapes were obtained. These results could be used in iceform design of shapes for complex flows such as wing/fuselage and tube/fin junctures.

1. INTRODUCTION

THE COUETTE iceform design model [1] can be used as a basis for two- and three-dimensional shape design. The Couette iceform model applies to both the ice and water fields close to the interface. Examination of any flow (laminar, transitional or turbulent) close to a surface reveals that the velocity profile closely resembles a linear velocity profile. For example, a laminar boundary layer has a limiting velocity gradient which arises as the wall is approached as shown by Schlichting [2] and Paff [3]. The linear velocity profile close to the wall is also found in turbulent situations as supported by the concept of the laminar sublayer. In addition the turbulent boundary layer is represented by a universal velocity profile that yields a characteristic velocity on the outer fringe (characteristic wall coordinate) of the laminar sublayer.

In any case, the near wall region at any point can be approximated by a Couette flow provided a characteristic velocity and effective fluid layer thickness are defined. For example, Fig. 1 shows a linear extension of the wall shear stress in a wedge flow boundary layer. The thermal boundary layer is represented by Nusselt number characteristics at the surface along with Prandtl number, pressure gradient and starting length dependence.

1.1. *Interface point model*

The Couette model can be applied at any point along the ice/water interface boundary. This yields local momentum, heat transfer and energy dissipation behavior that is controlled by global flow and thermal parameters. Consequently, one interface point is used to solve for the optimal thermal parameters for a given Reynolds number. Since the ice formation process is being used as a natural design tool [4], the designer does not wish to control the whole interface shape; rather only gross features such as frontal area, maximum or minimum ice thickness are considered. A gross feature is described by one point on the ice/water interface.

The string analogy, double [5] or single [6], akin to the basic string problem in the Calculus of Variations can be used to visualize control of two- and three-dimensional ice geometry. The geometry is controlled at a point while the remaining geometry is determined by the natural physics. The point can be controlled such that the resulting shape minimizes an energy functional. Other end points are determined by natural boundary conditions rather than specifying zero geometric variation. An evolution theory outlined in the previous paper [1] is used to determine the thermal control parameters that yield minimum energy dissipation for a given Reynolds number.

1.2. *Motivation for flat plate iceformation example*

Iceformation design over a flat plate laminar flow is a good problem to show use of the Couette model and the evolution theory. Much is known about laminar flow and heat transfer over a flat plate (Blasius) and over wedge shapes (Falkner–Skan) [7]. The two-dimensional velocity and temperature behavior leads to similarity solutions where the square root of the Reynolds number is the flow control parameter as shown by Schlichting [2].

Second, previous work has been devoted to describing ice formation over a flat plate in laminar flow and different thermal boundary conditions. For example, Hirata *et al.* [8] report theoretical and experimental results for ice profiles and correlate answers to a heat transfer "product", the axial Reynolds number times the square of the thermal parameter. It was also demonstrated that a one-dimensional description suffices for values of the heat transfer product greater than 9

$$Re_x \theta_T^2 > 9. \quad (1)$$

NOMENCLATURE

b/δ	ice thickness root parameter
Br	Brinkman number
d/β_I	designation function
D	virtual cylinder diameter
F	correlation factor
g/θ_I	generation function
I	ice thickness over plate
m	velocity axial exponent
Nu_x	axial Nusselt number
Nu_ζ	water space Nusselt number
Pr	Prandtl number
Re_D	cylinder Reynolds number
Re_r	reference Reynolds number
Re_x	axial Reynolds number
Sl	starting length factor
x	axial coordinate
X	flow and thermal parameters
y_e	effective water space.

Greek symbols

β_W/β_I	energy dissipation strength ratio
δ	water channel height
δ_e	effective Couette plate spacing
ζ	I normalized by δ
η_e	effective wall coordinate
κ	thermal conductivity ratio
θ_T	thermal parameter
θ_W/θ_I	heat transfer strength ratio
ξ	starting length
Φ	energy dissipation
Φ/β_I	relative energy dissipation
ω	wedge angle from wall
Ω_0	zero evolution length root.

Subscripts

i	iterative steps
M	minimum energy dissipation
S	steady state.

Superscript

$\hat{}$	optimal condition.

This yields a guideline for selecting interface points that are likely to be represented well by the Couette iceform model. Seki *et al.* [9] show the result of flat plate iceformation when the free stream is accelerated by ice growing on an opposite wall. For certain parameter values, the laminar boundary layer transitions and a region of decreased ice thickness results due to turbulent heat transfer enhancement.

Thirdly, the flat plate geometry is of practical significance as an endwall in the juncture with cylinder and turbine blade bodies. Recent work by LaFleur and Langston [10] has explored the resulting ice interface contours from a cylinder/flat plate juncture with an oncoming laminar boundary layer. The three-dimensional juncture flow drag was reduced by an average of 18% using the iceformation method.

In this paper the iceformation design over a cooled flat plate is presented. The problem is modeled using a wedge solution and the Couette iceform model of the near interface flow and thermal fields in both the ice and water. A number of governing equations are summarized followed by solution of steady state ice thickness, steady and optimal ice profiles, and calculation of optimal thermal control parameters. The evolution theory is used to show development of generation function, designation function and energy

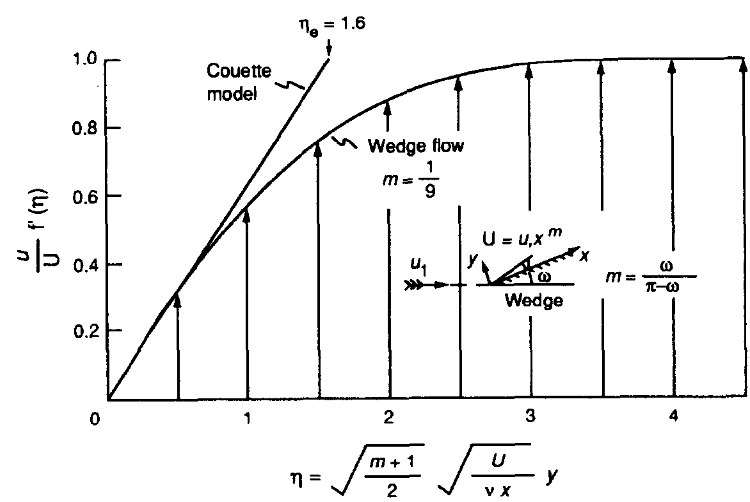

FIG. 1. Couette model of flow over a wedge.

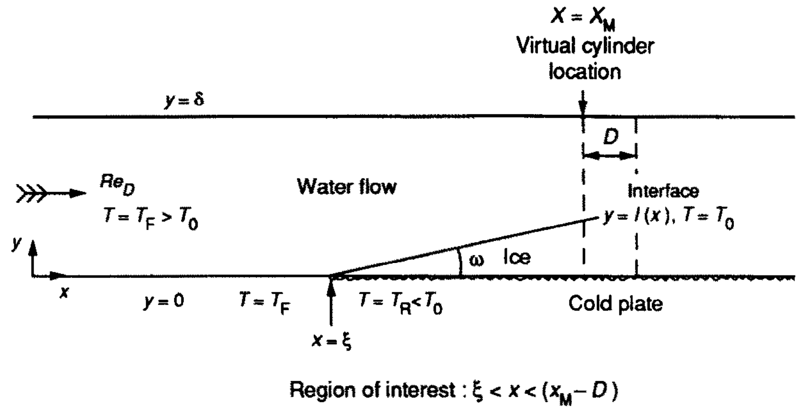

FIG. 2. Water channel for flat plate iceforms.

dissipation as steady state is reached. It is also shown that optimal geometries are adapted to parameters.

2. PROBLEM DEFINITION

The flat plate iceformation problem is produced by cooling the wall of a traditional Blasius type flat plate laminar boundary layer. The theoretical thermal boundary condition is introduced sharply at some distance downstream of the beginning of velocity boundary layer development; a starting length exists. Figure 2 shows the system configuration of a finite water channel with the freezing parent surface downstream of the boundary layer origin.

2.1. *Planning for later juncture study*

The information gained from the study of the flat plate can be used in juncture design problems. Figure 2 indicates the virtual location of a cylinder body that would be used to create a juncture. The specified Reynolds number is based on the fixed characteristic length of the cylinder diameter. However, this paper addresses only the flat plate condition.

The region of interest spans from the beginning of the cold plate ending one diameter upstream of the virtual cylinder. Turbulent juncture heat transfer data given by Ireland and Jones [11], and Goldstein *et al.* [12] indicate that the enhanced heat transfer in the juncture region does not extend appreciably beyond one diameter upstream of the cylinder leading edge. Heat transfer data is not available for the laminar juncture flow. However, Baker's landmark work [13] on the laminar horseshoe vortex shows that the three-dimensional juncture flow does not extend beyond one diameter upstream. Also the pressure gradients are less pronounced one diameter upstream of the cylinder obstruction. Therefore, the region of interest does not extend beyond this virtual axial location.

2.2. *Wedge model*

Previous investigators of the flat plate ice formation problem such as Hirata *et al.* [14] have shown that the ice profile resembles a wedge shape. The laminar case ice profile starts from zero thickness and monotonically thickens in the downstream direction. This is consistent with the decrease of Nusselt number as a thermal boundary layer develops. The interface equation or the steady state generation function indicate that a lower interface Nusselt number yields thicker ice over the parent surface, as shown in Fig. 2. The modeled wedge shape is expected to depart slightly from previously reported results due to variation in plate thermal-step boundary condition and due to approximations in the model.

The wedge shape is modeled using a Falkner–Skan type heat transfer solution coupled with a starting length factor given by Kays and Crawford [7]

$$Nu_x = \frac{F(Pr,m)Re_x^{1/2}}{Sl(x,\xi)} \tag{2}$$

where the starting length factor is given by equation (3)

$$Sl(x,\xi) = \left[1-\left(\frac{\xi}{x}\right)^{3/4}\right]^{1/3}. \tag{3}$$

The heat transfer coefficient for the wedge is obtained using a bilinear fit of Table 9-2 in Kays and Crawford as

$$F(Pr,m) = 0.52-0.297m+Pr[0.021+0.139m] \tag{4}$$

where the Prandtl number for water near freezing is 13.44 and where the wedge angle leads to free stream velocity as a power function of the axial position where the exponent is given by equation (5)

$$m = \frac{\omega}{\pi-\omega}. \tag{5}$$

The equivalent wedge angle is related to the ice thickness at an axial position relative to the starting length where the ice layer is assumed to begin

$$\omega = \arctan\left(\frac{I}{x-\xi}\right). \tag{6}$$

Additionally the Reynolds number based on the virtual cylinder diameter is linked to a reference Reynolds number based on the axial coordinate (with zero ice thickness)

$$Re_r = Re_D \frac{x}{D}. \tag{7}$$

The ice layer creates a narrowing of the channel water space; thus an acceleration of the "free stream" velocity is used to augment the reference Reynolds number based on axial position. The Reynolds number based on the axial coordinate with ice layer development is then

$$Re_x = Re_r \frac{\delta}{\delta - I} = Re_D \frac{x}{D} \frac{\delta}{\delta - I}. \tag{8}$$

The pressure gradient does not remain zero due to the ice layer development over the flat plate.

2.3. *Problem control parameters*

The iceformation design problem is controlled by two parameters: a flow parameter and a thermal parameter. The flat plate iceform is controlled by Reynolds number based on a fixed length (the virtual cylinder diameter instead of axial position or ice thickness) and the temperature ratio defined in the previous paper [1]. The two parameters represent the two degrees of freedom that exist in the design problem. Consequently, other quantities such as evolution length can be specified and the thermal or flow parameter can be solved. This type of approach addresses an adaptive design goal [6]. In summary, the adaptive design goal is to determine the conditions that lead to optimal geometries. The adaptive goal finds conditions or control parameters that satisfy

$$\left(\frac{\partial \zeta_s}{\partial X}\right) = 0 \quad \text{at } X = \hat{X} \tag{9}$$

where the independent variable may be a vector or scaler combination of parameters

$$X = X(Re_D, \theta_T). \tag{10}$$

This contrasts the traditional design optimization goal of determining the geometries that are optimal. The traditional goal finds geometries that satisfy

$$\left(\frac{\partial \Phi}{\partial \zeta}\right) = 0 \quad \text{at } \zeta = \zeta_M. \tag{11}$$

The Couette iceform model was formulated to satisfy the traditional design optimization goal. The Couette iceform model is used to represent the energy dissipation dependence on the flat plate ice geometry. Since the ice geometry adapts to the specified flow and thermal parameters, solution of the traditional goal leads to the determination of optimal control parameters.

Additional parameters are provided by arbitrary quantities that describe the scale of the boundaries. For example, the virtual cylinder diameter and location both contribute to defining the region of interest as shown in Fig. 2. The magnitude of the entry length determines the location of the ice wedge beginning. The channel width is an important parameter for modeling the velocity acceleration due to ice layer development. Therefore, the multi-dimensional iceformation problem has more than one characteristic length. The virtual cylinder diameter is the most-fixed characteristic length and is used for the Reynolds number flow parameter.

3. STEADY STATE CHARACTERIZATION

The evolution theory utilizes dynamic variation equations and a thermodynamic selection criterion. The first solution procedure determines the steady ice geometries in terms of prescribed flow and thermal parameters. Steady state is governed by a set of wedge model and Couette iceform model equations and an iterative algorithm.

3.1. *Applying the Couette model*

The Couette model represents simplification of both the flow and thermal fields in both the ice and water phases about a region close to the interface. The equivalent Couette iceform of the flat plate can be determined by examining the wedge model. Figure 1 shows a similarity solution for the velocity profile over a wedge shape. The outer boundary layer velocity is chosen as the characteristic velocity. This yields an equivalent Couette flow layer that is a fraction of the true boundary layer thickness. The result is an equivalent water space that is stated in terms of Reynolds number based on the axial position, the axial position and an equivalent similarity variable

$$y_e = \eta_e \sqrt{\left(\frac{2}{m+1}\right)} x Re_x^{-1/2}. \tag{12}$$

Based on the previous flat plate studies [5, 8, 9, 14], the wedge angle is expected to be small. Therefore the equivalent similarity variable is approximately 1.6 corresponding to a nominal axial power of velocity of 1/9 as shown in Fig. 1. This yields an equivalent water space of

$$y_e = 2.16x\, Re_x^{-1/2}. \tag{13}$$

The effective Couette plate spacing is the effective water space plus the ice thickness

$$\delta_e = y_e + I. \tag{14}$$

The Couette model of the water space thermal field is stated in terms of Nusselt number based on the equivalent water space [1]. This Nusselt number is related to the local Nusselt number model based on axial position over the assumed wedge shape

$$Nu_\zeta = Nu_x \frac{y_e}{x}. \tag{15}$$

Equations (13), (8) and (2)–(6) are combined into equation (15) to calculate the Nusselt number based on effective water space given the appropriate inputs or implicitly

$$Nu_\zeta = Nu_\zeta(I, Re_D, Pr, x, \xi, D, \delta). \tag{16}$$

The Couette model for the ice phase applies to the temperature field. The model is applied at a point one diameter upstream of the virtual cylinder location. Based on the work of Hirata *et al.* [8], equation (1) indicates that a one-dimensional model will work well. Also based on the assumption of small wedge angle, axial conduction in the ice layer is expected to be small. Conduction in a small wedge ice layer over a flat plate is largely one-dimensional which is precisely the Couette ice condition. The curved leading edge of the ice layer may be a difficult area for the Couette ice model to represent.

3.2. *Numerical scheme*

The calculation of the steady state iceform can be accomplished by three different methods, namely:

(1) transient simulation using the interface equation;
(2) numerical iteration satisfying steady state conditions; and
(3) hybrid—forcing transient steps toward steady state.

In all cases, the path of calculation appears as a geometric transient. Only method (1) is fully physical because real time is involved. Method (2) searches for convergence of the geometric iteration. Method (3) is useful to perform real time simulation when the heat transfer strengths depend on geometry in a nonlinear fashion.

In this paper, method (2) is used because the heat transfer strengths depend on geometry and the transient behavior is not considered. The steady state condition is input as a Couette model formula, zero generation rate or

$$I_{i+1} = \frac{y_e + I_i}{1 + \dfrac{\theta_W}{\theta_I}} \tag{17}$$

where the heat transfer strength ratio depends on the ice geometry through its dependence on the water space Nusselt number or

$$\frac{\theta_W}{\theta_I}(I) = \kappa\theta_T Nu_\zeta(I, \ldots). \tag{18}$$

The Nusselt number is then related, by the wedge model equations given above, to ice thickness, Prandtl number, starting length, axial position, wedge angle

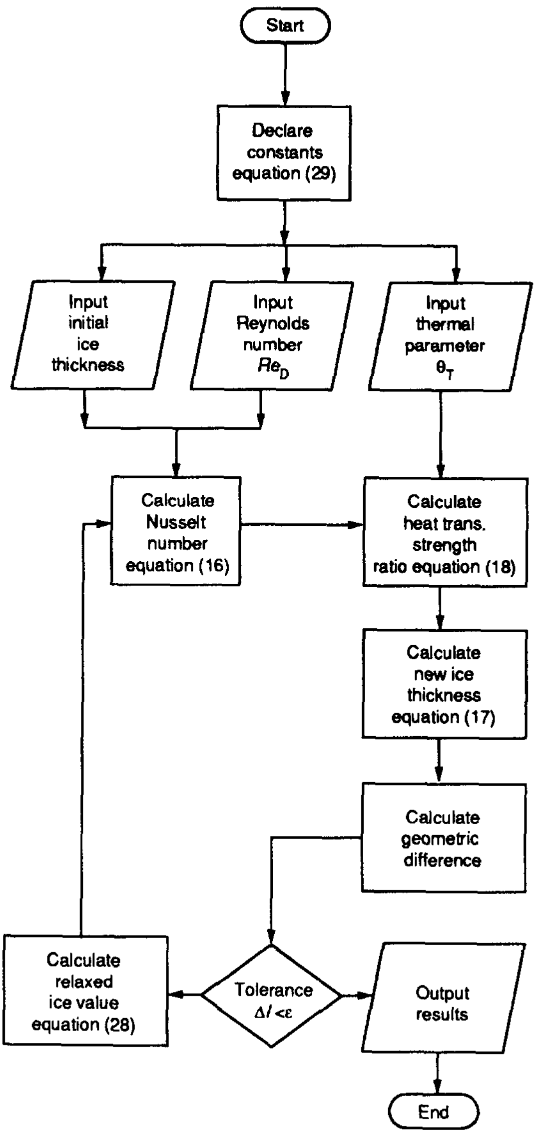

FIG. 3. Steady state algorithm.

and the equivalent local Reynolds number. Since the equivalent water space and local Reynolds number depend on the ice thickness an eigenvalue problem arises. The eigenvalue problem is solved by iteration between the Couette model condition of steady state and the matching of the wedge model heat transfer that is required for the particular ice thickness. Figure 3 provides a flowchart of the steady state solution algorithm. Convergence of the ice geometry solution is checked by using a tolerance level between succeeding iteration steps.

4. MINIMUM ENERGY DISSIPATION

The ice geometry thermodynamically influences the thermal and mechanical energy dissipation of the ice and water fields. In the previous paper [1] the evolution theory for the Couette iceform design model related the energy dissipation and an optimal selection criterion for a region close to the interface. The Couette iceform model is now used to represent the energy dissipation behavior of the ice shape. This provides a local functional for the iceform design performance.

For a flat plate boundary layer flow, the entropy production or energy dissipation is concentrated in the near wall region. For example, Bejan [15] reviewed the viscous dissipation calculation in turbulent boundary layers and showed that 'the wall region plays the dominant role in the production of entropy'. Similarly, laminar boundary layer flows are characterized by the wall region velocity gradient, i.e. friction factor and shear work.

4.1. *Couette model of energy dissipation*

When both thermal and flow energy dissipation are considered, they compete and a convex performance functional arises. Then a minimum entropy production or minimum energy dissipation ice geometry can be found. The Couette model is used to represent the dominant thermal and flow energy dissipation terms in the near wall region. From the Couette iceform model, the Brinkman number is related to the surface Nusselt number based on the water space as

$$Nu_\zeta = 1+\frac{Br}{2}. \tag{19}$$

The minimum energy dissipation condition has been formulated by the Couette model to be zero evolution length [1]. This yields a specific value for the Brinkman number as a function of the thermal conductivity ratio

$$\hat{Br} = \frac{2\Omega_0\sqrt{\frac{3}{4}}}{1-\Omega_0\sqrt{\frac{3}{4}}} \tag{20}$$

where the optimum value of omega is determined by zero evolution length to be

$$\Omega_0 = \sqrt{(1-\kappa)}. \tag{21}$$

Consequently the optimal Nusselt number based on the effective water space is

$$\hat{Nu}_\zeta = 1+\frac{\hat{Br}}{2}. \tag{22}$$

For the ice and water Couette iceform model,

$$\kappa = 0.27, \quad \Omega_0 = 0.85,$$

$$\hat{Br} = 5.69 \quad \text{and} \quad \hat{Nu}_\zeta = 3.85. \tag{23}$$

When both the minimum energy dissipation criterion and the Reynolds number are selected, the problem degrees of freedom is zero. Then the thermal parameter is solved and the optimal thermal parameter for that given Reynolds number is obtained. The thermal parameter is solved by inverting the previous steady state type equations. This results in a quasi-steady solution as

$$\hat{\theta}_\Gamma = \frac{1}{\kappa}\frac{1}{\hat{Nu}_\zeta}\frac{\theta_W}{\theta_I} \tag{24}$$

where the optimal Nusselt number is stated above and the heat transfer strength ratio, given by the steady state condition, is the ratio between the Couette iceform model water space and the ice thickness

$$\frac{\theta_W}{\theta_I} = \frac{y_e}{I}. \tag{25}$$

4.2. *Numerical scheme*

The numerical scheme for calculating the optimal thermal parameter and the optimal steady state geometry is presented in the algorithm of Fig. 4. An iteration is required for two reasons: firstly, the non-

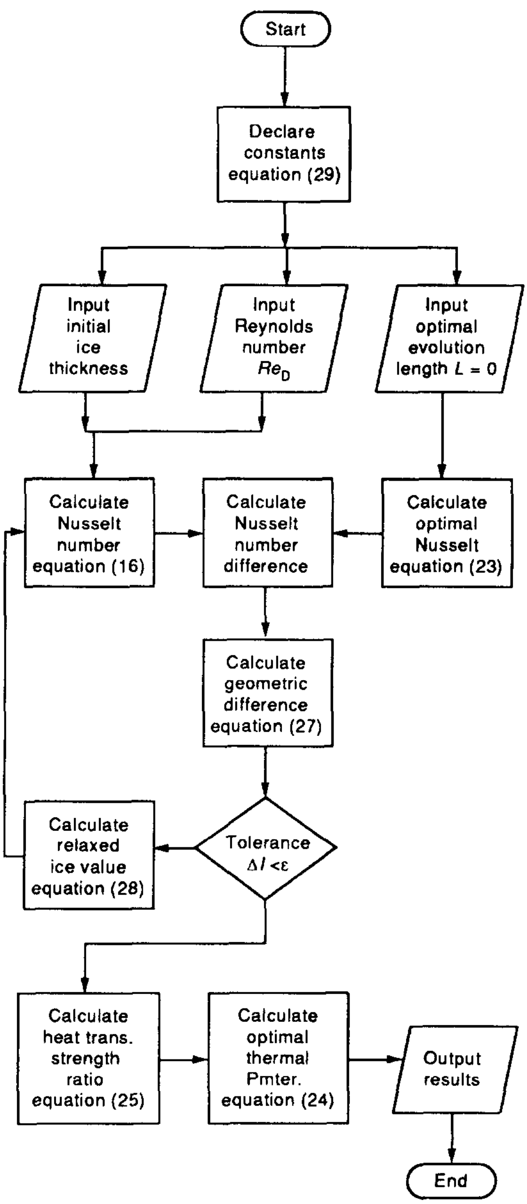

Fig. 4. Optimal steady state algorithm.

linear dependence of the wedge model Nusselt number on ice geometry and secondly, the indeterminance of selecting zero evolution length and Reynolds number in the Couette iceform model as discussed by LaFleur [1].

The iteration is tracked by comparing the optimal Nusselt number to the Nusselt number from the wedge model. This yields a difference between the optimal Nusselt number and the interface Nusselt number

$$\Delta Nu_{\zeta} = Nu_{\zeta} - \hat{N}u_{\zeta}. \tag{26}$$

The geometric change is guided by the difference in these Nusselt numbers and a relaxation factor of 0.5 is used to control the iteration

$$\Delta I = -\frac{1}{10}\Delta Nu_{\zeta} \tag{27}$$

and

$$I_{i+1} = I_i + 0.5\Delta I. \tag{28}$$

The convergence is measured by a tolerance of the geometric change.

5. RESULTS AND DISCUSSION

The steady state ice geometry and minimum energy dissipation calculations were performed using the previously defined wedge model and Couette iceform model formulas and the evolution algorithms stated in Figs. 3 and 4. The steady state ice geometry calculation resulted in ice geometry profiles, generation function, designation function and energy dissipation developments for prescribed Reynolds number and thermal parameters. The minimum energy dissipation calculation resulted in optimal thermal parameter distributions, zero evolution length and the identification of a parametric relationship between Reynolds number and the optimal thermal parameter.

5.1. *Steady state modeling results*

The steady state ice geometry for the flat plate iceformation was calculated by solving an eigenvalue problem in an iterative fashion. The ice geometry development one diameter upstream of the virtual cylinder location was tracked by the generation and designation functions. The behavior of the model was tested by using combinations of Reynolds number and the thermal parameter coupled with experimental apparatus parameters as follows:

$$D = 5.08\,\text{cm}, \quad x = 63.5\,\text{cm},$$
$$\xi = 54.1\,\text{cm} \quad \text{and} \quad \delta = 13.4\,\text{cm}. \tag{29}$$

Figure 5 shows geometric iterations vs the generation function, calculated by

$$\frac{g}{\theta_{\text{I}}} = \frac{\delta_c - I}{\delta_c} - \frac{I}{\delta_c}\frac{\theta_{\text{W}}}{\theta_{\text{I}}} \tag{30}$$

where the heat transfer strength ratio is given by equation (10). A nonphysical transient arises due to dissatisfaction of the steady state condition in the eigenvalue problem. The steady state solution is characterized by a single control parameter, a heat transfer product based on equation (10) of the form

$$X = Re_{\text{D}}\theta_{\text{T}}^2. \tag{31}$$

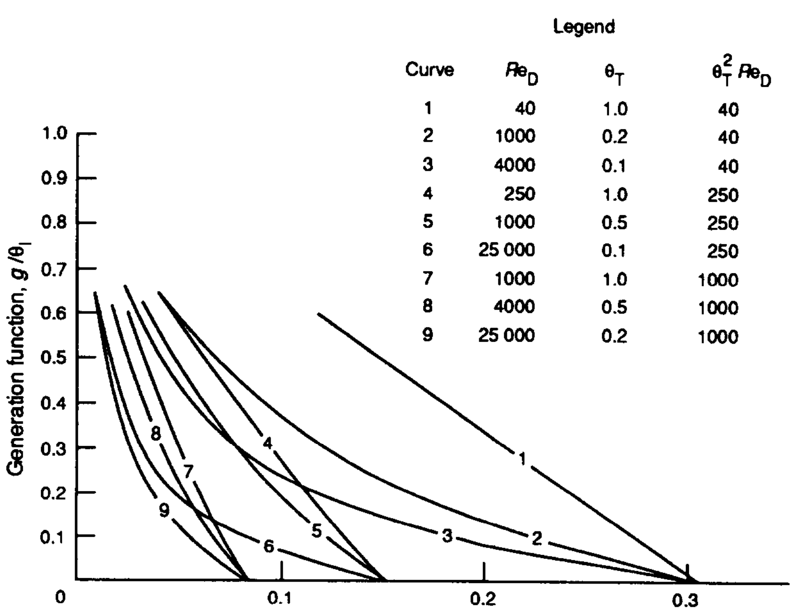

FIG. 5. Generation function vs ice thickness and flow and thermal parameters.

Table 1. Test cases for the Couette iceform model applied to flat plate iceformation

Authors	Case #	δ (cm)	ξ (cm)	x (cm)	Re_D	θ_T	I_{dat} (cm)	I_{calc} (cm)
Hirata *et al.* (1979), laminar	1	40.6	0	20.0	3637	0.105	6.0	5.69
	2	40.6	0	16.5	1560	0.330	3.2	2.82
	3	40.6	0	21.5	1560	0.625	2.0	1.81
Hirata *et al.* (1979), near transition	4	40.6	0	34.0	22 124	0.175	1.6	2.18
	5	40.6	0	30.0	22 386	0.185	1.3	1.97
	6	40.6	0	29.0	21 840	0.204	1.4	1.75
Seki *et al.* (1984)	7	1.5	300	320	9313	0.300	1.0	1.12
	8	1.5	300	320	18 627	0.286	0.8	0.98
LaFleur (1988)	9	13.3	54	63.5	1843	0.44	1.3	1.76
	10	13.3	54	63.5	1843	0.37	1.7	2.03
	11	13.3	54	63.5	1843	0.27	2.1	2.62

The three example values of steady state geometry differ by different values of the heat transfer product. Figure 5 shows the examples of 40, 250 and 1000. Although the steady state is characterized by the single parameter, the iterative path to steady state depends separately on the flow and thermal parameters. This is shown by the different paths converging on the generation function axis at the same ice thickness. The different paths also show varying degrees of non-linearity in the Couette iceform and wedge models of the flat plate iceformation. However, for the cases shown (ice growth) the steady state solutions are stable.

The Couette iceform model was also tested for cases cited in the literature. Table 1 indicates a variety of situations investigated by ice formation researchers [8, 14, 9, 5]. Flat plate studies that indicated different channel geometries, flow speeds and thermal parameter values accompanied by ice thickness data were used to test the robustness of the Couette iceform model for the flat plate. Since the current wedge model is for laminar flow, only cases that used a subcritical Reynolds number were considered, i.e. $Re_D < 24\,000$. The algorithm of the Couette model was run using the stated experimental conditions. The model ice thicknesses can then be compared to the experimental data. Figure 6 shows a plot of experimental and Couette model ice thicknesses. The $x = y$ line indicates the relative success of the Couette model. Deviations result from three possible effects:

(1) free convection effects in the boundary layer due to maximum density instability of water at 4°C

(2) starting length approximation and fuzzy thermal boundary condition at the cold plate edge

(3) non-monotonic velocity profiles in the accelerating boundary layer over the wedge-ramp.

These effects are not specifically addressed in the present model. The free convection effect can be checked by calculating the Grashof number for the most unstable water temperature difference as was done for laminar flat plate ice formation by Hirata *et al.* [8]. In the cases cited in Table 1, the Grashof number was well below 200; thus the free convection effects were small. However, in some cases longitudinal ridges along the ice wedge were observed. This means that free convection effects still occur even for subcritical Grashof numbers. This demonstrates the sensitivity of the ice interface shape to flow pattern variations.

The origin of the ice layer on the flat plate was found to occur very close to the cold thermal boundary condition. In the wedge model the thermal step is sharp. In experiments of ice formation over a finite thickness flat plate, the thermal step is not sharp. This leads to variations in the actual thermal boundary condition, the thermal boundary layer and the origin of the ice layer. This is especially true near the origin of the ice layer where the ice layer is highly curved and the one-dimensional conduction assumption is suspect. Figure 7 displays ice profiles over a cold flat plate for variations in the flow and thermal parameters. The Couette model is used over the entire

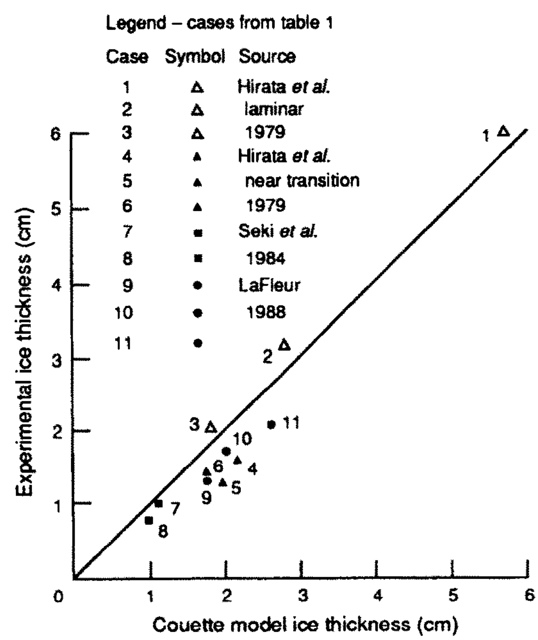

Fig. 6. Comparison of Couette model and experimental ice thicknesses from previous flat plate studies.

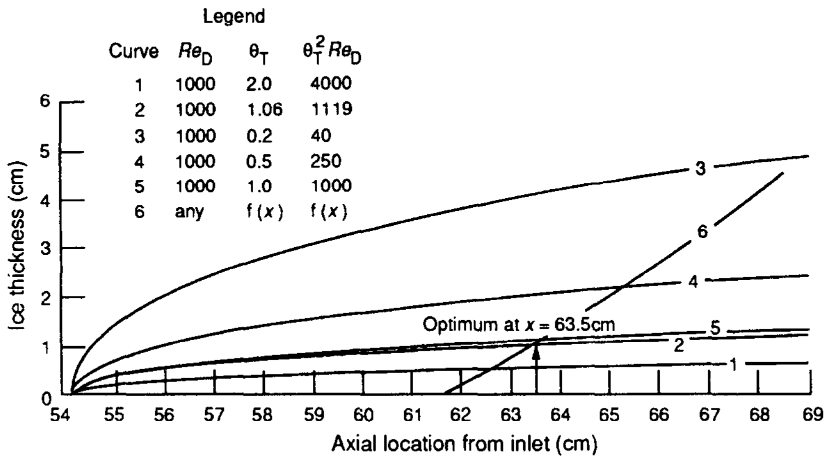

FIG. 7. Steady state ice layer profiles.

region of interest by changing the axial location in the steady state calculation. The profiles shown qualitatively match those found in studies cited in Table 1. Approximations in the ice layer origin region may lead to discrepancies in the relatively flat ice profile downstream.

The velocity profile accelerates up and over the origin of the ice layer. Hirata *et al.* [8] found that the velocity profile experiences an inflection point due to this acceleration. Thus the velocity profile is not monotonic as in the wedge type solutions given by Schlichting [2]. The inflection would lead to a different effective water space in the Couette iceform model and different ice thickness results.

In summary, the steady state calculation is an eigenvalue problem to solve the following equation:

$$\frac{I_s}{\delta} = \sqrt{\left(\left(\frac{b}{\delta}\right)^2 + 2\left(\frac{b}{\delta}\right)\right)} - \frac{b}{\delta} \tag{32}$$

where

$$\frac{b}{\delta} = \frac{1}{2\kappa^2[Re_D\theta_T^2]}\left(\frac{D}{\delta}\right)\left(\frac{x}{\delta}\right)\left(\frac{Sl(x,\xi)}{F(Pr,m(I))}\right)^2. \tag{33}$$

Equations (3) and (4) are used for starting length and wedge angle factors.

5.2. *Energy dissipation results*

The previous steady state results were extended by investigating the energy dissipation characteristics of the flat plate iceformation. This is relevant to demonstrate the potential of using iceformation to design flow and thermal shapes. The Couette model is best utilized for the purpose of approximating near-interface viscous and thermal dissipation. This provides a means of judging the use of iceformation as a flow and thermal design tool.

Figure 7 illustrates the solution of an optimal ice contour over a variable temperature plate. This yields a criterion for combinations of Reynolds number, axial position and thermal parameter that yield zero evolution length shapes, i.e. steady state ice geometries that minimize energy dissipation. For example, the optimal criterion at an axial location of 63.5 cm is given by

$$Re_D\theta_T^2 = 1119. \tag{34}$$

Consequently, there is only one optimal geometry at any axial location as shown in Fig. 7. A more general result for any axial position is found by calculating the optimal thermal parameter for given Reynolds number, axial position and water tunnel parameters. This utilizes the optimal Nusselt number value of 3.85 from equation (23) and matches the actual ice heat transfer by iteration of the ice position.

Figure 8 illustrates the evolution of ice geometry for various combinations of Reynolds number and the thermal parameter. The designation function is calculated by

$$\frac{d}{\beta_I} = \frac{\beta_W}{\beta_I}\left(\frac{I}{\delta_e}\right)^2 - \left(\frac{\delta_e - I}{\delta_e}\right)^2 \tag{35}$$

where the energy dissipation strength ratio is governed by the Couette iceform model as

$$\frac{\beta_W}{\beta_I} = \kappa\theta_T^2\left[1 + Br - \frac{Br^2}{12}\right] \tag{36}$$

and

$$Br = 2[Nu_\xi - 1]. \tag{37}$$

The evolution path is jagged due to large changes in the interface position at the beginning of evolution. As evolution proceeds the steps become smaller and the geometry improves on a smooth path. The evolution path ceases at steady state and does not necessarily yield zero evolution length (designation function at steady state). In two of the examples the flow and thermal parameters were chosen such that the evolution does not end in optimal steady states. One

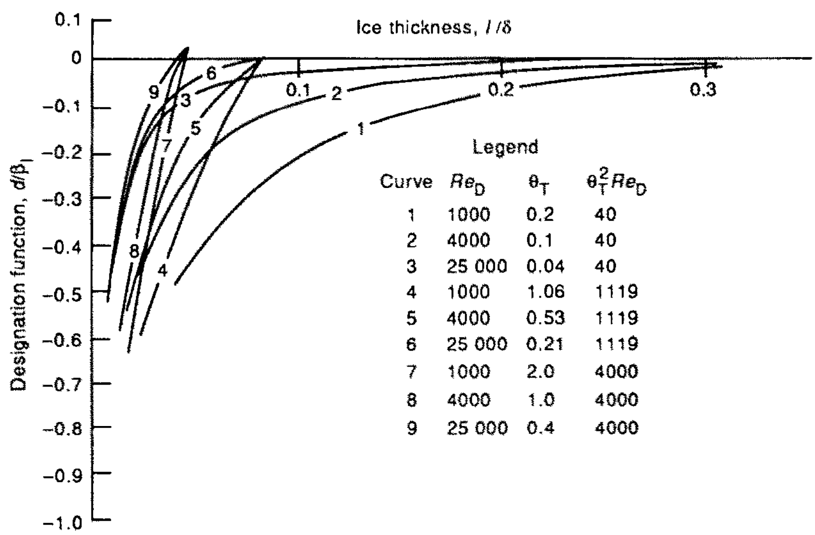

Fig. 8. Designation function vs ice thickness and flow and thermal parameters.

steady state has a positive evolution length and the other negative. The case that yields optimal steady state corresponds to the optimal thermal parameter as determined by equation (34). In all cases the evolution path is dependent separately on the Reynolds number and the thermal parameter. The flat plate Couette iceform model yields evolutions that are thermodynamically stable.

The designation function indicates the sensitivity of the energy dissipation to geometric variations. The convergence of designation function values at steady state do not correspond to convergence of ice geometry energy dissipation. Figure 9 illustrates this point. The same cases as shown in Fig. 8 are considered for energy dissipation. Again, the evolution paths are jagged at the start but become smooth as the geometry converges on steady state. All ice geometry evolutions led to a reduction in the energy dissipation as calculated by

$$\frac{\Phi}{\beta_l} = \frac{\delta_c}{l} + \frac{\beta_W}{\beta_l}\frac{\delta_c}{\delta_c - l} = \frac{y_c + l}{l} + \frac{\beta_W}{\beta_l}\frac{y_c + l}{y_c}. \tag{38}$$

However the near-minimum values of energy dissipation depend separately on Reynolds number and the thermal parameter. The curves in Fig. 9 show

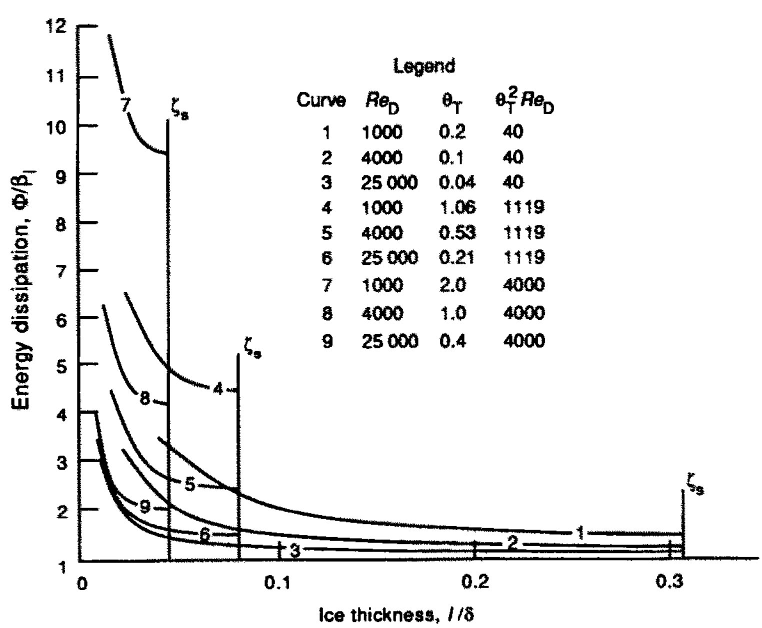

Fig. 9. Energy dissipation vs ice thickness and flow and thermal parameters.

that a higher thermal parameter yields a thinner ice thickness and higher energy dissipation. Similarly, a higher Reynolds number yields a thinner ice thickness but nearly the same energy dissipation. This is due to counteracting Reynolds number terms in the wedge model, effective water space and Couette model.

In summary, the energy dissipation characteristics of the flat plate iceformation can be approximated by the Couette iceform model. Minimum energy dissipation at steady state is obtained by using the optimal Nusselt number value in the heat transfer and ice thickness equations

$$\frac{Sl(x,\xi)}{F(Pr,m)} = \frac{\eta_e}{\hat{N}u_\xi}. \tag{39}$$

This expression can be substituted into equation (33) to obtain an eigenvalue equation for the optimal steady state geometry

$$\frac{\hat{\delta}}{\delta} = \frac{1}{2\kappa^2[Re_D\theta_T^2]}\left(\frac{D}{\delta}\right)\left(\frac{x}{\delta}\right)\left(\frac{\eta_e}{\hat{N}u_\xi}\right)^2 \tag{40}$$

where equation (39) must be matched by the interface heat transfer at steady state.

6. CONCLUSIONS

The Couette iceform model was used to investigate two-dimensional iceformation over a cold flat plate. Steady state ice profiles were calculated by using different axial locations. The Couette model was tested using information from previous flat plate ice formation investigations. The Couette model was successful at predicting point ice thicknesses and ice profile shapes. Steady state ice thickness was found to be correlated with a heat transfer product: the Reynolds number multiplied by the square of the thermal parameter.

The Couette iceform model was used to calculate the energy dissipation characteristics of the flat plate iceformation. The optimal criterion of zero evolution length was used to determine values of the optimal thermal parameter for a given Reynolds number. The optimal criterion led to the heat transfer product being specifically equal to a function of the axial position and the 'unnel geometry parameters. In general, steady state geometries did not minimize energy dissipation. Energy dissipation was reduced during the ice geometry iteration.

The flat plate example showed the utility of the Couette model to approximate two-dimensional steady state ice geometries and energy dissipation. The Couette model can be used to control the iceformation process as a shape design tool by selection of flow and thermal parameters. The correlation of results with Reynolds number and the thermal parameter allow the preprocess calculation of conditions for iceformation design experiments. Couette iceform modeling of different iceformation regimes and development of correction factors are topics for further investigation.

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EXEMPLE DU MODELE DE DESSIN DE FORME DE GLACE SELON COUETTE: FORMATION DE GLACE SUR PLAQUE PLANE

Résumé—On présente une application du modèle de contour de formation de glace selon Couette. La formation de glace bidimensionnelle sur une plaque plane froide est considérée pour démontrer l'approche par la théorie d'évolution pour contrôler le mécanisme de formation de glace. Les profils théoriques de glace s'accordent raisonnablement bien avec les résultats expérimentaux. Des expressions du paramètre de contrôle optimal sont obtenues pour dessiner des formes à dissipation minimale d'énergie. Ces résultats peuvent être utilisés dans le dessin des formes de glaciation pour des écoulements complexes tels que les jonctions aile/fuselage et tube/ailette.

BEISPIEL FÜR DAS COUETTE EISBILDUNGSMODELL: EISBILDUNG AN EINER EBENEN PLATTE

Zusammenfassung—In dieser Arbeit wird eine Anwendung des Couette Eisbildungsmodells vorgestellt. Anhand der zweidimensionalen Eisbildung an einer kalten ebenen Platte wird die Anwendung der Evolutionstheorie auf die Beeinflussung des Formgebungsvorganges durch Eisbildung gezeigt. Die theoretischen Eisprofile stimmen mit experimentell ermittelten weitgehend überein. Es werden die Beziehungen für optimale Wahl der Einflußgrößen ermittelt, die zu einer Eisform mit minimaler Dissipationsenergie führt. Diese Ergebnisse können verwendet werden, um mit Hilfe der Eisbildung geometrische Formen für komplizierte Strömungen zu entwerfen, wie z. B. den Ansatzbereich einer Tragfläche oder den Rippenansatz an einem Rohr.

ИСПОЛЬЗОВАНИЕ МОДЕЛИ КУЭТТА ПРИ ЛЬДООБРАЗОВАНИИ НА ПЛОСКОЙ ПЛАСТИНЕ

Аннотация—Описывается применение модели Куэтта при образовании льда. Использование эволюционной теории для регулирования процесса формирования образующегося льда иллюстрируется на примере двумерного льдообразования на холодной плоской пластине. Теоретические профили слоя льда удовлетворительно согласуются с экспериментальными данными. Получены соотношения для параметров оптимального регулирования, соответствующих формам с минимальным рассеянием энергии. Результаты могут использоваться для расчета форм образующегося льда при сложных течениях, например, в случаях соединений крыло-фюзеляж и труба-ребро.